Exercises

3-A. Write each answer with a reasonable number of figures. Find the absolute and percent relative uncertainty for each answer.

(a) $[12.41\ (\pm0.09) \div 4.16\ (\pm0.01)] \times 7.068\ 2\ (\pm0.000\ 4) = ?$

(b) $[3.26\ (\pm0.10) \times 8.47\ (\pm0.05)] - 0.18\ (\pm0.06) = ?$

(c) $6.843\ (\pm0.008) \times 10^4 \div [2.09\ (\pm0.04) - 1.63\ (\pm0.01)] = ?$

(d) $\sqrt{3.24\ \pm\ 0.08} = ?$

(e) $(3.24 \pm 0.08)^4 = ?$

(f) $\log(3.24 \pm 0.08) = ?$

(g) $10^{3.24\ \pm\ 0.08} = ?$

3-B. (a) How many milliliters of 53.4 ($\pm$0.4) wt% NaOH with a density of 1.52 ($\pm$0.01) g/mL will you need to prepare 2.000 L of 0.169 M NaOH?

(b) If the uncertainty in delivering NaOH is $\pm$0.01 mL, calculate the absolute uncertainty in the molarity (0.169 M). Assume there is negligible uncertainty in the formula mass of NaOH and in the final volume (2.000 L).

3-C. We have a 37.0 ($\pm$0.5) wt% HCl solution with a density of 1.18 ($\pm$0.01) g/mL. To deliver 0.050 0 mol of HCl requires 4.18 mL of solution. If the uncertainty that can be tolerated in 0.050 0 mol is $\pm$2%, how big can the absolute uncertainty in 4.18 mL be? (*Caution:* In this problem, you have to work backward. You would normally compute the uncertainty in mol HCl from the uncertainty in volume:

$$\text{mol HCl} = \dfrac{\text{mL solution} \times \dfrac{\text{g solution}}{\text{mL solution}} \times \dfrac{\text{g HCl}}{\text{g solution}}}{\dfrac{\text{g HCl}}{\text{mol HCl}}}$$

But, in this case, we know the uncertainty in mol HCl (2%) and we need to find what uncertainty in mL solution leads to that 2% uncertainty. The arithmetic has the form $a = b \times c \times d$, for which $\%e_a^2 = \%e_b^2 + \%e_c^2 + \%e_d^2$. If we know $\%e_a$, $\%e_c$, and $\%e_d$, we can find $\%e_b$ by subtraction: $\%e_b^2 = \%e_a^2 - \%e_c^2 - \%e_d^2$.)

3-D. Compute the molecular mass and its standard uncertainty for NH_3. What is the percent relative uncertainty in molecular mass?

Problems

Significant Figures

3-1. How many significant figures are there in the following numbers?

(a) 1.903 0   **(b)** 0.039 10   **(c)** 1.40×10^4

3-2. Round each number as indicated:

(a) 1.236 7 to 4 significant figures

(b) 1.238 4 to 4 significant figures

(c) 0.135 2 to 3 significant figures

(d) 2.051 to 2 significant figures

(e) 2.005 0 to 3 significant figures

3-3. Round each number to three significant figures:

(a) 0.216 74   **(b)** 0.216 5   **(c)** 0.216 500 3

3-4. *Vernier scale*. The figure below shows a scale found on instruments such as a micrometer caliper used for accurately measuring dimensions of objects. The lower scale slides along the upper scale and is used to interpolate between the markings on the upper scale. In (*a*), the reading (at the left-hand 0 of the lower scale) is between 1.4 and 1.5 on the upper scale. To find the exact reading, observe which mark on the lower scale is aligned with a mark on the upper scale. Because the 6 on the lower scale is aligned with the upper scale, the correct reading is 1.46. Write the correct readings in (*b*) and (*c*) and indicate how many significant figures are in each reading.

3-5. Write each answer with the correct number of digits.

(a) $1.021 + 2.69 = 3.711$

(b) $12.3 - 1.63 = 10.67$

(c) $4.34 \times 9.2 = 39.928$

(d) $0.060\ 2 \div (2.113 \times 10^4) = 2.849\ 03 \times 10^{-6}$

(e) $\log(4.218 \times 10^{12}) = ?$

(f) antilog $(-3.22) = ?$

(g) $10^{2.384} = ?$

3-6. Write the formula mass of **(a)** BaF_2 and **(b)** $C_6H_4O_4$ with a reasonable number of digits. Use the periodic table inside the cover of this book to find atomic masses.
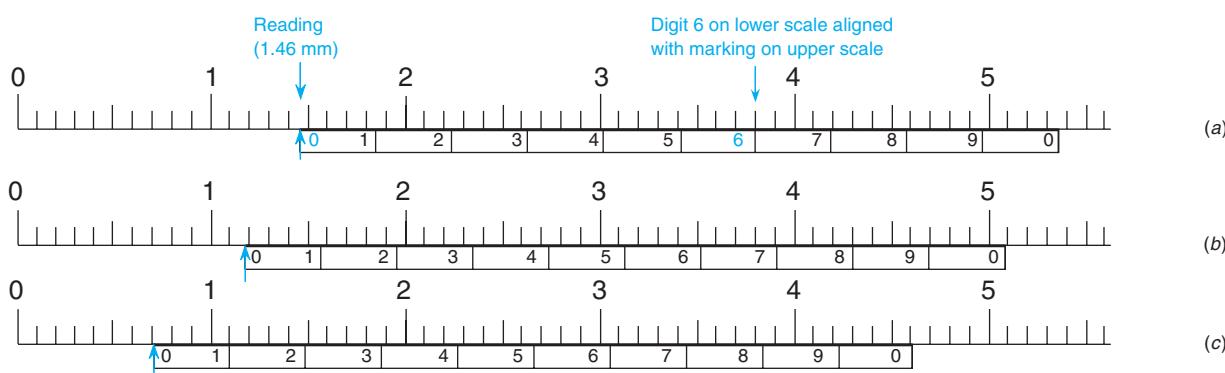

Figure for Problem 3-4.

3-7. Write each answer with the correct number of significant figures.

(a) $1.0 + 2.1 + 3.4 + 5.8 = 12.300\ 0$

(b) $106.9 - 31.4 = 75.500\ 0$

(c) $107.868 - (2.113 \times 10^2) + (5.623 \times 10^3) = 5\ 519.568$

(d) $(26.14/37.62) \times 4.38 = 3.043\ 413$

(e) $(26.14/(37.62 \times 10^8)) \times (4.38 \times 10^{-2}) = 3.043\ 413 \times 10^{-10}$

(f) $(26.14/3.38) + 4.2 = 11.933\ 7$

(g) $\log(3.98 \times 10^4) = 4.599\ 9$

(h) $10^{-6.31} = 4.897\ 79 \times 10^{-7}$

3-8. 📖 *Controlling the appearance of a graph.* Figure 3-3 requires gridlines to read buret corrections. In this exercise, you will format a graph so that it looks like Figure 3-3. Follow the procedure in Section 2-11 to graph the data in the following table. For Excel 2007, insert a Chart of the type Scatter with data points connected by straight lines. Delete the legend and title. With Chart Tools, Layout, Axis Titles, add labels for both axes. Click any number on the abscissa (*x*-axis) and go to Chart Tools, Format. In Format Selection, Axis Options, choose Minimum = 0, Maximum = 50, Major unit = 10, and Minor unit = 1. In Format Selection, Number, choose Number and set Decimal places = 0. In a similar manner, set the ordinate (*y*-axis) to run from −0.04 to +0.05 with a Major unit of 0.02 and a Minor unit of 0.01. To display gridlines, go to Chart Tools, Layout, and Gridlines. For Primary Horizontal Gridlines, select Major & Minor Gridlines. For Primary Vertical Gridlines, select Major & Minor Gridlines. To move *x*-axis labels from the middle of the chart to the bottom, click a number on the *y*-axis (not the *x*-axis) and select Chart Tools, Layout, Format Selection. In Axis Options, choose Horizontal axis crosses Axis value and type in −0.04. Close the Format Axis window and your graph should look like Figure 3-3.

For earlier versions of Excel, select Chart Type XY (Scatter) showing data points connected by straight lines. Double click the *x*-axis and select the Scale tab. Set Minimum = 0, Maximum = 50, Major unit = 10, and Minor unit = 1. Select the Number tab and highlight Number. Set Decimal places = 0. In a similar manner, set the ordinate to run from −0.04 to +0.05 with a Major unit of 0.02 and a Minor unit of 0.01, as in Figure 3-3. The spreadsheet might overrule you. Continue to reset the limits you want and click OK each time until the graph looks the way you intend. To add gridlines, click inside the graph, go to the Chart menu and select Chart Options. Select the Gridlines tab and check both sets of Major gridlines and Minor gridlines and click OK. In Chart Options, select the Legend tab and deselect Show legend. To move *x*-axis numbers to the bottom of the chart, double click the *y*-axis (not the *x*-axis) and

select the Scale tab. Set "Value (x) axis crosses at" to −0.04. Click OK and your graph should look like Figure 3-3.

Volume (mL)	Correction (mL)
0.03	0.00
10.04	0.04
20.03	0.02
29.98	−0.03
40.00	0.00
49.97	0.03

Types of Error

3-9. Why do we use quotation marks around the word *true* in the statement that accuracy refers to how close a measured value is to the "true" value?

3-10. Explain the difference between systematic and random error.

3-11. Suppose that in a gravimetric analysis, you forget to dry the filter crucibles before collecting precipitate. After filtering the product, you dry the product and crucible thoroughly before weighing them. Is the apparent mass of product always high or always low? Is the error in mass systematic or random?

3-12. State whether the errors in **(a)**–**(d)** are random or systematic:

(a) A 25-mL transfer pipet consistently delivers 25.031 ± 0.009 mL.

(b) A 10-mL buret consistently delivers 1.98 ± 0.01 mL when drained from exactly 0 to exactly 2 mL and consistently delivers 2.03 mL $\pm$ 0.02 mL when drained from 2 to 4 mL.

(c) A 10-mL buret delivered 1.983 9 g of water when drained from exactly 0.00 to 2.00 mL. The next time I delivered water from the 0.00 to the 2.00 mL mark, the delivered mass was 1.990 0 g.

(d) Four consecutive 20.0-μL injections of a solution into a chromatograph were made and the area of a particular peak was 4 383, 4 410, 4 401, and 4 390 units.

3-13. Cheryl, Cynthia, Carmen, and Chastity shot the targets below at Girl Scout camp. Match each target with the proper description.

(a) accurate and precise

(b) accurate but not precise

(c) precise but not accurate

(d) neither precise nor accurate

3-14. Rewrite the number 3.123 56 ($\pm$0.167 89%) in the forms **(a)** number ($\pm$ absolute uncertainty) and **(b)** number ($\pm$ percent relative uncertainty) with an appropriate number of digits.
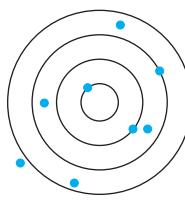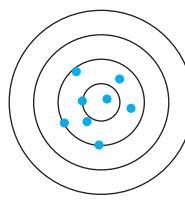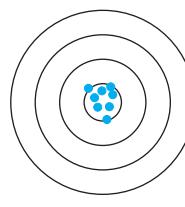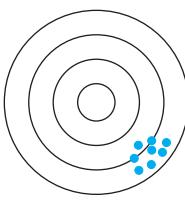
Cheryl   Cynthia   Carmen   Chastity

Figure for Problem 3-13.

Propagation of Uncertainty

3-15. Find the absolute and percent relative uncertainty and express each answer with a reasonable number of significant figures.

(a) $6.2\ (\pm 0.2) - 4.1\ (\pm 0.1) = ?$

(b) $9.43\ (\pm 0.05) \times 0.016\ (\pm 0.001) = ?$

(c) $[6.2\ (\pm 0.2) - 4.1\ (\pm 0.1)] \div 9.43\ (\pm 0.05) = ?$

(d) $9.43\ (\pm 0.05) \times \{[6.2\ (\pm 0.2) \times 10^{-3}] + [4.1\ (\pm 0.1) \times 10^{-3}]\} = ?$

3-16. Find the absolute and percent relative uncertainty and express each answer with a reasonable number of significant figures.

(a) $9.23\ (\pm 0.03) + 4.21\ (\pm 0.02) - 3.26\ (\pm 0.06) = ?$

(b) $91.3\ (\pm 1.0) \times 40.3\ (\pm 0.2)/21.1\ (\pm 0.2) = ?$

(c) $[4.97\ (\pm 0.05) - 1.86\ (\pm 0.01)]/21.1\ (\pm 0.2) = ?$

(d) $2.016\ 4\ (\pm 0.000\ 8) + 1.233\ (\pm 0.002) + 4.61\ (\pm 0.01) = ?$

(e) $2.016\ 4\ (\pm 0.000\ 8) \times 10^3 + 1.233\ (\pm 0.002) \times 10^2 + 4.61\ (\pm 0.01) \times 10^1 = ?$

(f) $[3.14\ (\pm 0.05)]^{1/3} = ?$

(g) $\log[3.14\ (\pm 0.05)] = ?$

3-17. Verify the following calculations:

(a) $\sqrt{3.141\ 5(\pm 0.001\ 1)} = 1.772\ 4_3\ (\pm 0.000\ 3_1)$

(b) $\log[3.141\ 5\ (\pm 0.001\ 1)] = 0.497\ 1_4\ (\pm 0.000\ 1_5)$

(c) $\text{antilog}[3.141\ 5\ (\pm 0.001\ 1)] = 1.385_2\ (\pm 0.003_5) \times 10^3$

(d) $\ln[3.141\ 5\ (\pm 0.001\ 1)] = 1.144\ 7_0\ (\pm 0.000\ 3_5)$

(e) $\log\left(\dfrac{\sqrt{0.104(\pm 0.006)}}{0.051\ 1(\pm 0.000\ 9)}\right) = 0.80_0\ (\pm 0.01_5)$

3-18. (a) Show that the formula mass of NaCl is $58.443\ (\pm 0.001_2)$ g/mol.

(b) To prepare a solution of NaCl, you weigh out $2.634\ (\pm 0.002)$ g and dissolve it in a volumetric flask whose volume is $100.00\ (\pm 0.08)$ mL. Express the molarity of the solution, along with its uncertainty, with an appropriate number of digits.

3-19. What is the true mass of water weighed at 24°C in the air if the apparent mass is $1.034\ 6 \pm 0.000\ 2$ g? The density of air is $0.001\ 2 \pm 0.000\ 1$ g/mL and the density of balance weights is 8.0 ± 0.5 g/mL. The uncertainty in the density of water in Table 2-7 is negligible in comparison to the uncertainty in the density of air.

3-20. Twelve dietary iron tablets were analyzed by the gravimetric procedure in Section 1-4 and the final mass of Fe_2O_3 (FM 159.688) was $0.277_4 \pm 0.001_8$ g. Find the average mass of Fe per tablet. (Relative uncertainties in atomic masses are small compared with relative uncertainty in the mass of Fe_2O_3. Neglect uncertainties in atomic masses in this problem.)

3-21. We can measure the concentration of HCl solution by reaction with pure sodium carbonate: $2H^+ + Na_2CO_3 \rightarrow 2Na^+ + H_2O + CO_2$. Complete reaction with $0.967\ 4 \pm 0.000\ 9$ g of Na_2CO_3 (FM $105.988\ 4 \pm 0.000\ 7$) required 27.35 ± 0.04 mL of HCl. Find the molarity of the HCl and its absolute uncertainty.

3-22. Avogadro's number can be computed from the following measured properties of pure crystalline silicon:[4] (1) atomic mass (obtained from the mass and abundance of each isotope), (2) density of the crystal, (3) size of the unit cell (the smallest repeating unit in the crystal), and (4) number of atoms in the unit cell. For silicon, the mass is $m_{Si} = 28.085\ 384\ 2\ (35)$ g/mol, where 35 is the uncertainty (standard deviation) in the last two digits. The density is $\rho = 2.329\ 031\ 9\ (18)$ g/cm^3, the size of the cubic unit cell is $c_0 = 5.431\ 020\ 36\ (33) \times 10^{-8}$ cm, and there are 8 atoms per unit cell. Avogadro's number is computed from the equation

$$N_A = \frac{m_{Si}}{(\rho c_0^3)/8}$$

From the measured properties and their uncertainties, compute Avogadro's number and its uncertainty. To find the uncertainty of c_0^3, use the function $y = x^a$ in Table 3-1.

3-23. Express the molecular mass ($\pm$uncertainty) of $C_9H_9O_6N_3$ with the correct number of significant figures.

3-24. *Estimating uncertainty in Keeling's CO_2 measurements*. The manometer in Box 3-2 has the following key uncertainties:

Volume of large vessel: $5.013\ 8_2 \pm 0.000\ 5$ L
Volume of small vessel: $3.793\ 0 \pm 0.000\ 9$ mL
Pressure: ± 0.03 mm Hg in ~400 mm Hg
Temperature: ± 0.03 K in ~300 K

Which factor has the largest percent relative uncertainty? If one uncertainty is sufficiently larger than the others, then the uncertainty in measuring CO_2 is governed by that factor. Considering just the most uncertain factor, estimate the uncertainty in 400 ppm CO_2 in air.